

# Quarkonium Spectral Function from Anisotropic Lattice

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## References

Work in collaboration with **A. Jakovac** (Budapest, Tech. U.),  
**P. Petreczky** (Brookhaven) and **K. Petrov** (Bohr Inst.).

- **hep-lat/0603005**
- **PoS LAT2005:153,2006 (hep-lat/0509138)**

# Outline

## 1 Introduction

- Meson Correlators and Spectral Functions
- Reconstruction of the Spectral Function
- Simulation parameters, lattices

## 2 Charmonium

- Zero Temperature
- Finite Temperature

## 3 Bottomonium

## 4 Summary

## Point meson operator

$$J_H(t, x) = \bar{q}(t, x)\Gamma_H q(t, x),$$

where  $\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \gamma_\mu\gamma_\nu$ .

Meson states in different channels:

$\Gamma$	$^{2S+1}L_J$	$J^{PC}$	$c\bar{c}$ (n=1)	$c\bar{c}$ (n=2)	$b\bar{b}(n=1)$ (n=1)	$b\bar{b}(n=2)$ (n=2)
$\gamma_5$	$^1S_0$	$0^{-+}$	$\eta_c$	$\eta'_c$	$\eta_b$	$\eta'_b$
$\gamma_s$	$^3S_1$	$1^{--}$	$J/\psi$	$\psi'$	$\Upsilon(1S)$	$\Upsilon(2S)$
$\gamma_s\gamma_{s'}$	$^1P_1$	$1^{+-}$	$h_c$		$h_b$	
1	$^3P_0$	$0^{++}$	$\chi_{c0}$		$\chi_{b0}(1P)$	$\chi_{b0}(2S)$
$\gamma_5\gamma_s$	$^3P_1$	$1^{++}$	$\chi_{c1}$		$\chi_{b1}(1P)$	$\chi_{b1}(2P)$
		$2^{++}$	$\chi_{c2}$		$\chi_{b2}(1P)$	$\chi_{b2}(2P)$

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## The spectral function

$$\sigma_H(p_0, \vec{p}) = \frac{1}{2\pi} (D_H^>(p_0, \vec{p}) - D_H^<(p_0, \vec{p})) = \frac{1}{\pi} \text{Im} D_H^R(p_0, \vec{p})$$

$$D_H^{>(<)}(p_0, \vec{p}) = \int \frac{d^4 p}{(2\pi)^4} e^{ip.x} D_H^{>(<)}(x_0, \vec{x}) \quad (1)$$

$$D_H^>(x_0, \vec{x}) = \langle J_H(x_0, \vec{x}), J_H(0, \vec{0}) \rangle$$

$$D_H^<(x_0, \vec{x}) = \langle J_H(0, \vec{0}), J_H(x_0, \vec{x}) \rangle, x_0 > 0 \quad (2)$$

The Euclidean propagator

$$G_H(\tau, \vec{p}) = \int d^3 x e^{i\vec{p}.\vec{x}} \langle T_\tau J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle$$

is related to the spectral function through the integral representation

$$G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

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# Reconstruction of the Spectral Function

- $G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma(\omega, T) K(\omega, \tau, T)$
- $\mathcal{O}(10)$  data and  $\mathcal{O}(100)$  degrees of freedom to reconstruct.
- Bayesian technique: find  $\sigma(\omega, T)$  that maximizes  $P[\sigma|DH]$ .
  - $D$  data
  - $H$  prior knowledge:  $\sigma(\omega, T) > 0$

**Maximum Entropy Method:** Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH] = \exp\left(-\frac{1}{2}\chi^2 + \alpha S\right) \quad (3)$$

Shannon-Janes entropy:  $S = \int d\omega \left[ \sigma(\omega) - m(\omega) - \sigma(\omega) \ln\left(\frac{\sigma(\omega)}{m(\omega)}\right) \right]$ ,  
 $m(\omega)$  - the default model,  $m(\omega \gg \Lambda_{QCD}) = m_0 \omega^2$  - perturbation theory.

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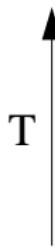
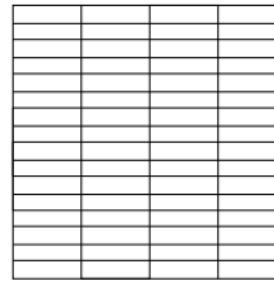
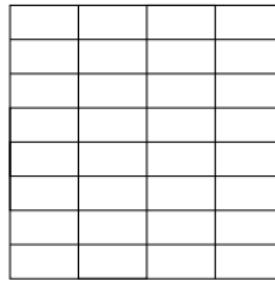
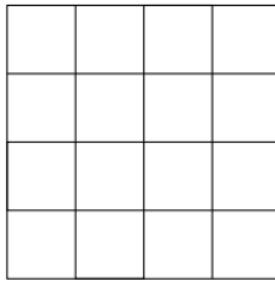
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Anisotropic lattice  $\xi = a_s/a_\tau = 2$  and 4.



Standard Wilson action in the gauge sector and the anisotropic clover improved action for heavy fermions. Quenched approximation.  
Sommer scale to fix the physical units.

$\beta$	5.7	5.9	6.1	6.1	6.5
$N_s^2 \times N_t$	$8^3 \times 64$	$16^3 \times 64$	$16^3 \times 64$	$16^3 \times 96$	$24^3 \times 160$
$(\xi, \xi_0)$	(2,1.655)	(2,1.691)	(2,1.718)	(4,3.211)	(4,3.3166)
$r_0/a_s$	2.414(8)	3.690(11)	5.207(29)	5.189(21)	8.96(4)
$a_t^{-1} [\text{Gev}]$	1.905	2.913	4.110	8.181	14.12
$L_s [\text{fm}]$	1.66	2.17	1.54	1.54	1.34
configs	2000	1560	930	500	160

Table: Simulation parameters for charmonium at zero temperature.

# Charmonium: $T = 0$

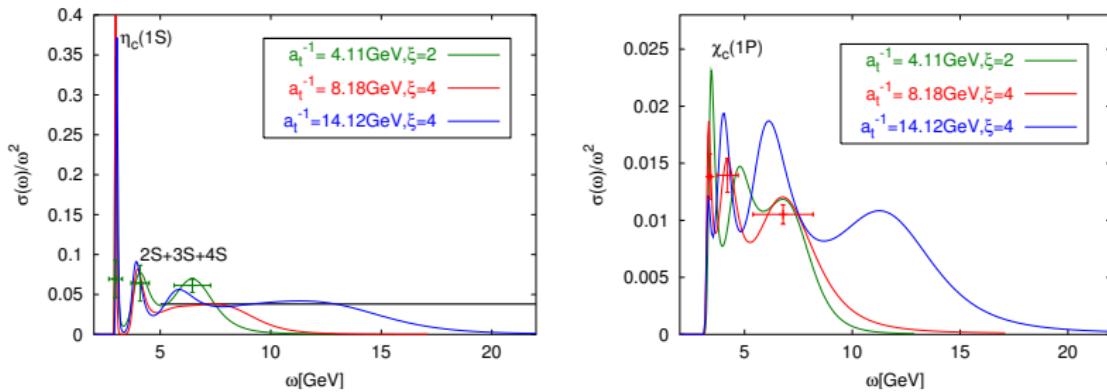
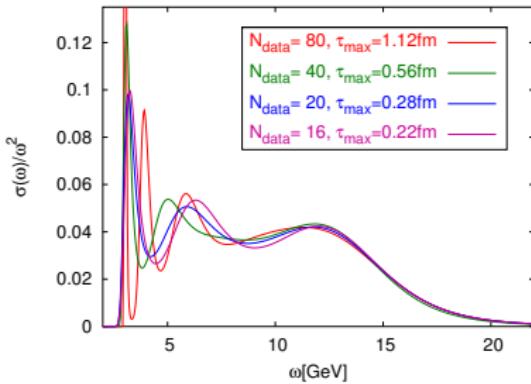
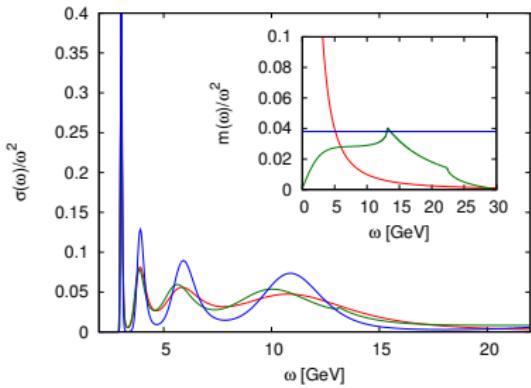


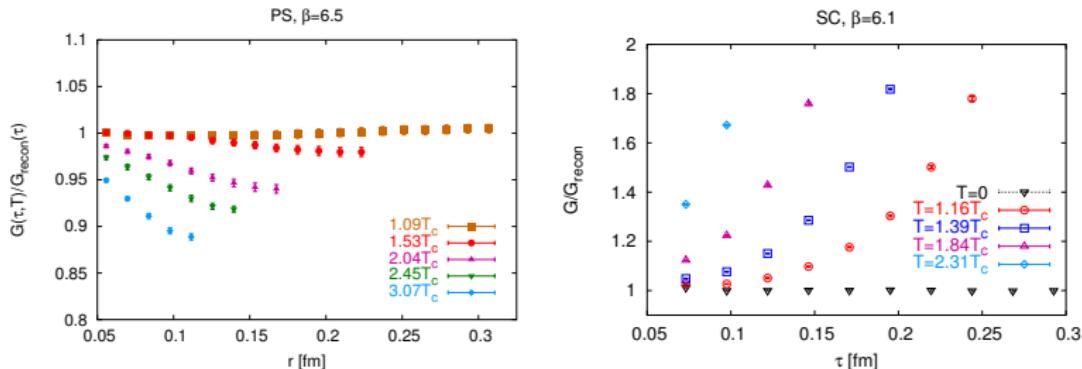
Figure: Charmonium spectral function in the pseudoscalar channel (left) and the scalar channel (right) at different lattice spacings and zero temperature.



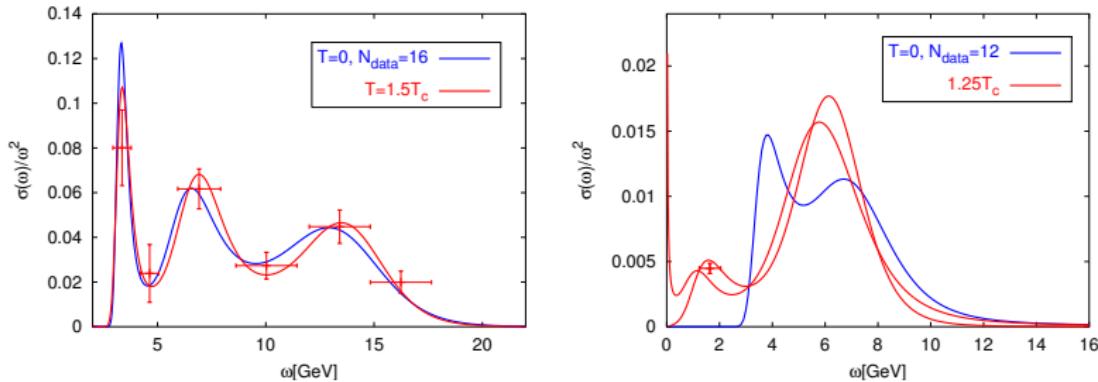
**Figure:** Charmonium spectral function dependence on the default model (left) and on the maximum time extend (right). Pseudoscalar channel at  $a_t^{-1} = 14.12 \text{GeV}$  and zero temperature.

Charmonium:  $T > 0$ 

$$G_{\text{recon}}(\tau, T) = \int_0^{\infty} d\omega \sigma(\omega, T=0) K(\tau, \omega, T)$$



**Figure:** The ratio  $G(\tau, T)/G_{\text{recon}}(\tau, T)$  of charmonium for pseudoscalar channel at  $a_t^{-2} = 14.11 \text{ GeV}$  (left) and scalar channel at  $a_t^{-2} = 8.18 \text{ GeV}$  (right) at different temperatures.



**Figure:** Charmonium spectral function in the pseudoscalar channel at  $a_t^{-2} = 14.11 \text{ GeV}$  (left) and the scalar channel (right) at  $a_t^{-2} = 8.18 \text{ GeV}$  at zero and above deconfinement temperatures. For finite temperature scalar channel two different default models are shown.

# Bottomonium

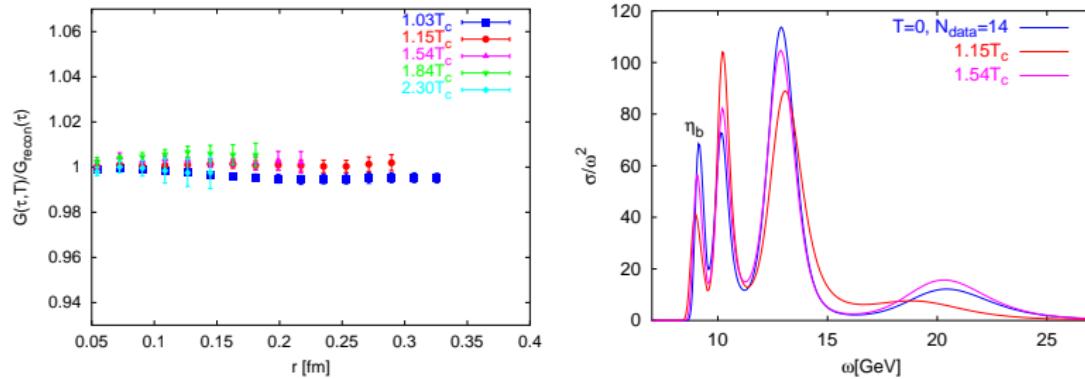
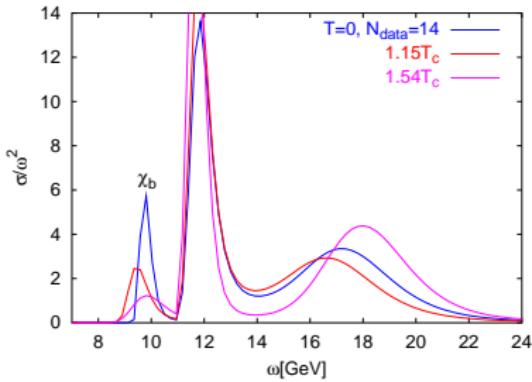
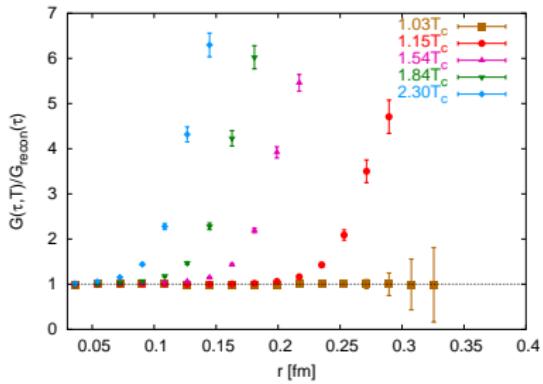


Figure: Bottomonia correlators (left) and spectral functions (right) in pseudo-scalar channel for different temperatures.



**Figure:** Bottomonia correlators (left) spectral functions (right) in scalar channel for different temperatures.

# Summary

- The  $1S$  ( $\eta_c$ ,  $J/\psi$ ) charmonium states **exist** as a resonance in the deconfined phase at  $T \simeq 1.5 T_c$ .
- $1P$  ( $\xi_{c0}$ ,  $\xi_{c1}$ ) charmonium states **dissolve** at  $1.1 T_c$ .
- Bottomonium states show similar behavior. Melting of  $1P$  bottomonia states was **unexpected**.
- The emerging studies of heavy quarkonium properties with dynamical quarks produce consistent results with the quenched approximation results.

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